Exercise 10F – Worked Solutions to the Problems

Hints

1. Assume that water is flowing into a narrow container at the same rate as it is flowing into a wide container. In which container is the water rising faster?

2. Look at the gradient at a typical point on the curve – is it positive or negative?

   Look at how the gradient is changing as we move along the curve from left to right – is it increasing or decreasing.

3. Recall that the area under a velocity-time graph gives displacement.

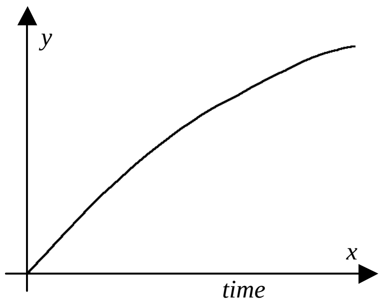
   Recall that distance is unsigned displacement.

4. Recall the method you used in Exercise 10D, Questions 6 and 7 to find the value of the gradient at a number of points on a given function. Then try to determine the function that passes through those points.

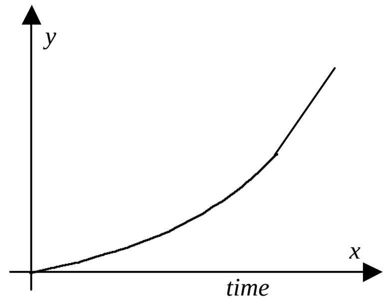
Exercise 10F – Worked Solutions to the Problems

1. Note: the graphs are sketches only, they are not exact.

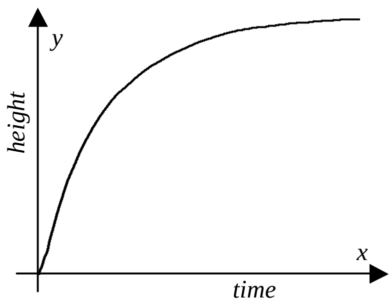
a.



b.



c.



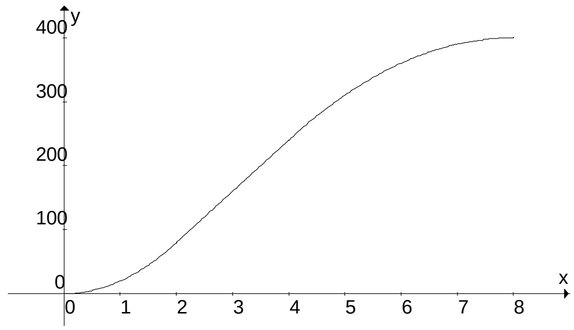
2. a. At time $t = 0$, both the distance and the velocity (i.e. the gradient) are zero. Further the gradient is positive and increasing as time increases. This could be the graph of the displacement of a car accelerating from a standing start (e.g. at a stoplight).

   b. At time $t = 0$, the distance is a positive value. At time $t = 0$, the velocity is zero. The velocity is increasingly negative as time increases. This could be the graph of the displacement of a car which is some distance away, and which is accelerating towards the starting point.

   c. At time $t = 0$, the distance is a positive value. At time $t = 0$, the velocity is a large negative value. As time increases, the gradient approaches zero. This could be the graph of a the displacement of a car which is some distance away, initially travelling towards the starting point at high speed, but gradually braking. It comes to a stop some distance away from the starting point.

   d. At time $t = 0$, the distance is zero and the velocity is positive. However, as time goes on the velocity gradually slows to zero. This could be the graph of the displacement of a car which initially is travelling away at high speed, but which gradually brakes to a halt.

3.



The total displacement is best found by finding the area under the velocity-time graph.

From 0 to 2 seconds:	$A = \frac{1}{2}bh = \frac{1}{2} \times 2 \times 80$	= 80 units^2
From 2 to 4 seconds:	$A = lw = 2 \times 80$	= 160 units^2
From 4 to 8 seconds:	$A = \frac{1}{2}bh = \frac{1}{2} \times 4 \times 80$	= 160 units^2
TOTAL:		**= 400 units^2**

4. Using the methods of Exercise 10D, Question 6, first construct a table of gradients for different values of *x*:

x	***gradient m***
-2	-0.1875
-1	-3
-0.5	-48
0	-
.5	-48
1	-3
2	-0.1875

Now we need to find a function that gives these values. First note that it is an even function, since $f(-x) = f(x)$. Also note that the function is not defined for $x = 0$, and the values for $x = -0.5$ and $x = 0.5$ imply that this may well be a vertical asymptote. The values of the gradient are all negative.

All of the above imply that the function may be of the form: $m = -\frac{a}{x^{2n}}$.

Since $m = -3$ when $x = 1$, this implies that $a = 3$. Some trial and error (or some algebra) will show that: $m = -\frac{3}{x^4}$ fits the table perfectly.

Alternatively, put the following data into List1 and List2 on your graphics calculator:

List1	***List2***
.5	48
1	3
2	0.1875

{Note: we had to use positive values, because the logarithms of negative numbers are not defined. We will put the negative sign back later.)

Applying power regression, shows us that the function that fits this data is $y = 3x^{-4}$ and hence the function for the original data is $y = -3x^{-4}$, which agrees with our solution above.