Mathematics For Queensland
Year 11 Mathematics B
Chapter Two – Quadratic Functions

Additional Problems

Note: Most of these questions are very challenging for Year 11 Mathematics B students. These problems require good understanding and/or a high level of algebraic skill. Even solving ONE of them is reason for celebration.

1. You have learned many different methods of solving quadratic equations - describe each of them. State the strengths and weaknesses of each method. Identify the one you prefer and explain why you prefer it.

2. It takes a very high initial velocity to keep a thrown ball in the air for seven seconds – try it! The record time in the air for a thrown baseball is about eight seconds. Assuming a baseball is in the air for eight seconds, and the force of gravity, a = -10 ms^{-2}:

   a. what is the initial vertical velocity?

   b. what is the maximum height reached?

   c. The quadratic formula used to answer the above questions can be thought of as a mathematical model for the height of a projectile at a given time. What assumptions are made when we use this model?

   d. How realistic are these assumptions? Or, put another way, how might the mathematical model be modified to give a more accurate model?

3. You are managing a holiday complex that contains 100 units. From past records you estimate that if you charge $600 per unit per week, all units will be rented over the coming holiday season. However for each increase of $50 per week, 5 additional units will become vacant. What weekly cost will maximise the **revenue** (i.e. the total collected, before expenses)?

4. Here is a paradox that involves completing the square. The conclusion is obviously incorrect. What is the error in this argument?

$$16-36=25-45$$
$$4^2-9^2=5^2-9\times5$$
$$4^2-9\times4+\frac{81}{4}=5^2-9\times5+\frac{81}{4}$$
$$(4-\frac{9}{2})^2=(5-\frac{9}{2})^2$$
$$4-\frac{9}{2}=5-\frac{9}{2}$$
$$4=5$$

5. The triangular numbers are 1, 3, 6, 10, 15, 21 … . These numbers form a **sequence** of **terms**. The first triangular number is designated t_1, the second is designated t_2, and so on. The general term (i.e. the n^{th} term), is designated t_n.

   a. By substitution, confirm that the formula for the n^{th} triangular number is given by

   $$t_n = \frac{n(n+1)}{2}$$

   b. Prove the following: Given any three consecutive triangular numbers, the sum of the middle number with the product of the first and third numbers is equal to the square of the middle numbers.

6. a. Confirm that these are Pythagorean Triads: 3,4,5; 5,12,13; 7,24,25; 9,40,41

   b. Confirm that the sequence above can be generated with the n^{th} set of triplets (x, y, z) being $x = 2n+1,\ y = 2n^2+2n,\ z = 2n^2+2n+1$.

   c. Find the next three Pythagorean Triads in this sequence.

   d. Prove that if we plot x against y, all points lie on the graph of $y = \dfrac{x^2-1}{2}$

   e. Prove that the distance between the origin and any point (x, y) on the graph of $y = \dfrac{x^2-1}{2}$ is given by z.

7. Solve the equation $\dfrac{x}{\frac{x-1}{x-2}} = x-3$, rounded off to two decimal places.

8. If the equation $ax^2+bx+c=0$ has two real number solutions $x = x_1$ and $x = x_2$, write an equation (using a, b and c) whose solutions are $-x_1$ and $-x_2$. Justify your solutions.

9. An algebra class was asked to solve a quadratic equation of the form $x^2+px+q=0$. Two students miscopied the problem - one miscopying only the coefficient of x, and the other miscopying only the constant term. The first found the answers to be 2 and 4. The second found the answers to be 2 and 7.

   If both solved their equations correctly, find the original equation and the correct roots.

10. A student solved the equation $(11-x)(x-4)=6$ as follows: $11-x=6$ or $x-4=6$. Hence $x=5$ or $x=6$. The student then came up with another equation, $(6-x)(x-9)=-4$, which he solved in a similar way: $6-x=-4$ or $x-9=-4$ and hence $x=10$ or $x=5$.

    In both cases, the answers are correct. Find **all** the equations that can be solved in this way. Justify your solution.

11. a. A square is inscribed in a right triangle with sides 3, 4 and 5 units. Find the area of the square.

b. A square is inscribed in a right triangle with sides a, b and c units. Find the area of the square (in terms of a, b and c).