Exercise 7J – Worked Solutions to the Problems

Hints

1. Recall that $32 = 16 \times 2$, and $a^5 = a^4 \times a$.

2. Recall that $\frac{1}{4} = \left(\frac{1}{2}\right)^2$.

3. Recall the change of base rule: $\log_a b = \dfrac{\log_c b}{\log_a b}$.

4. Recall that $P = P_0 \times a^t$. What are P_0, P and a in this problem? Take the log of both sides to solve for t.

5. Given a fixed annual rate of growth, the population is given by $P = P_0 \times a^t$, where P_0 is the population at time t_0, and a is the annual rate of growth. Solve these resulting equation for t.

6. Recall Log Law 2: $\log A - \log B = \log \dfrac{A}{B}$. In words, "The difference of two logs equals the log of their quotient." How might this fact be useful for finding an answer to the problem posed?

7. Note that $1025 = 4100 \div 2^2$ and that $1024 = 2^{10}$. Use this information to write an expression for a involving log 4100 and log 2.

8. Use the change of base rule to write each of the logarithms to a common base, say 10. Then use log laws and other algebra to simplify.

9. Let $2^{\log_2 3} = x$ and re-write this index equation as a log equation. First note that $9 = 3^2$, so change the base to 3. Now set this expression equal to y and re-write this index equation as a log equation.

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1. $$\begin{aligned}\sqrt[4]{32a^5} &= \sqrt[4]{16\times 2\times a^4\times a}\\ &= \sqrt[4]{16}\times\sqrt[4]{2}\times\sqrt[4]{a^4}\times\sqrt[4]{a}\\ &= 2\times a\times\sqrt[4]{2}\times\sqrt[4]{a}\\ &= 2a\sqrt[4]{2a}\end{aligned}$$

2. Recall: $\frac{1}{4}=\left(\frac{1}{2}\right)^2$.

$\therefore$ $$\begin{aligned}\left(\frac{1}{4}\right)^{\frac{1}{4}} &= \left[\left(\frac{1}{2}\right)^2\right]^{\frac{1}{4}}\\ &= \left(\frac{1}{2}\right)^{\frac{2}{4}}\\ &= \left(\frac{1}{2}\right)^{\frac{1}{2}}\end{aligned}$$

3. Now $$\begin{aligned}\log_c b^2 &= 2\log_c b\\ &= 2\frac{\log b}{\log c}\end{aligned}$$

And $$\begin{aligned}\log_b c^3 &= 3\log_b c\\ &= 3\frac{\log c}{\log b}\end{aligned}$$

$\therefore$ $$\begin{aligned}\log_c b^2\times\log_b c^3 &= 2\frac{\cancel{\log b}}{\cancel{\log c}}\times 3\frac{\cancel{\log c}}{\cancel{\log b}}\\ &= 6\end{aligned}$$

4. $P=P_0\times a^t$, where
P_0 = 500 000,
P = 500 000 000, and
a = 2.

Substituting $$\begin{aligned}500\,000\,000 &= 500\,000\times 2^t\\ 1000 &= 2^t\\ \log 1000 &= \log 2^t\\ \log 1000 &= t\log 2\\ t &= \frac{\log 1000}{\log 2}\\ t &\doteq 9.966\end{aligned}$$

Since this represents the number of 20 minute periods, the number of minutes ≈ 199 minutes.

5. $P = P_0 \times a^t$, where
P_0 = 19 million at time t_0 (1998)
P = 30 million at time t
a = 1.016

Substituting

$$30 = 19 \times 1.016^t$$
$$\frac{30}{19} = 1.016^t$$
$$\log\frac{30}{19} = \log 1.016^t$$
$$\log 30 - \log 19 = t \log 1.016$$
$$t = \frac{\log 30 - \log 19}{\log 1.016}$$
$$t \doteq 28.8$$

Since $t_0 = 0$ in 1998, Australia's population will reach 30 million in 2027, if it continues to grow at the same rate.

6. Say List1= {2, 4, 12, 24, 72, 144, 1440}

Since Log Law 2 states that $\log\frac{A}{B} = \log A - \log B$, finding the differences in the logarithms of the numbers in List1 is equivalent to finding the ratios of the numbers themselves. So:

Let List2 = log(List1)
And List3 = ΔList(List2) {Find the difference in the logs}
So List4 = 10^{List3} {The inverse operation of taking logs}

List4 contains the required ratios.

7. $$a = \log\frac{1025}{1024} = \log\frac{4100 \div 2^2}{2^{10}} = \log\frac{\frac{4100}{2^2}}{2^{10}}$$

∴ $$a = \log\frac{4100}{2^{12}} = \log 4100 - \log 2^{12} = \log 4100 - 12\log 2$$

But $b = \log 2$

So $a = \log 4100 - 12b$

Hence $\log 4100 = a + 12b$

8. Using the change of base rule,

$$\log_x xyz = \frac{\log xyz}{\log x}, \quad \log_y xyz = \frac{\log xyz}{\log y}, \text{ and } \log_z xyz = \frac{\log xyz}{\log z}$$

So $\dfrac{1}{\log_x xyz} = \dfrac{1}{\frac{\log xyz}{\log x}} = \dfrac{\log x}{\log xyz}$ and similarly for the other two terms.

Hence
$$\frac{1}{\log_x xyz} + \frac{1}{\log_y xyz} + \frac{1}{\log_z xyz} = \frac{\log x}{\log xyz} + \frac{\log y}{\log xyz} + \frac{\log z}{\log xyz}$$

$$= \frac{\log x + \log y + \log z}{\log xyz} \quad \text{\{adding numerators\}}$$

$$= \frac{\log xyz}{\log xyz} \quad \text{\{Log Law 1\}}$$

$$= 1$$

9.

Let	$2^{\log_2 3} = x$	
Then	$\log_2 x = \log_2 3$	{rewrite the index equation as a log equation}
So	$x = 3$	{if $\log_c A = \log_c B$ then $A = B$}
Similarly	$9^{\log_3 2} = y$	
So	$3^{2\log_3 2} = y$	{write 9 as a power of 3}
	$3^{\log_3 2^2} = y$	{Log Law 3}
	$3^{\log_3 4} = y$	
Hence	$\log_3 y = \log_3 4$	{rewrite the index equation as a log equation}
So	$y = 4$	{if $\log_c A = \log_c B$ then $A = B$}
Finally	$2^{\log_2 3} + 9^{\log_3 2} = x + y$	
	$= 3 + 4$	
	$= 7$	