Exercise 8I – Worked Solutions to the Problems

Hints

1. Once you have taken out a factor of x, you can repeat the process on the expression inside the brackets.

2. Create a new function which is the difference between the cosine function and the polynomial. Use your graphics calculator to find when this difference reaches a maximum.

3. Sketch the functions $V(t)$ and $y = 100$ on the same set of axes.

4. Draw a large clear diagram. Find an expression for each length in the diagram. Also note that there are two similar triangles and that these triangles are right-angled. Apply your knowledge of similar triangles and Pythagoras' Theorem to the problem.

5. The pattern should be clear. Once you have found the expression for the amount of money in the bank at the end of the fifth year, use your graphics calculator to find x, and hence i.

Exercise 8I – Worked Solutions to the Problems

1. a. $f(x) = 2x^3 + 5x^2 - 6x + 3$

$= x(2x^2 + 5x - 6) + 3$

$= x[x(2x + 5) - 6] + 3$

b.

original expression:	$2x^3$	3 multiplications x 10 =	30 time units
	$5x^2$	2 multiplications x 10 =	20 time units
	$6x$	1 multiplication x 10 =	10 time units
		3 additions and subtractions =	3 time units
	Total		**63 time units**

factored expression	x[x(2x + 5) + 6] + 3		
		3 multiplications x 10 =	30 time units
		3 additions =	3 time units
	Total		**33 time units**

The factored expression takes a bit over half the time to evaluate.

c. $f(x) = 5x^4 - 4x^3 + 3x^2 - 2x + 1$

$= x(5x^3 - 4x^2 + 3x - 2) + 1$

$= x[x(5x^2 - 4x + 3) - 2] + 1$

$= x\{x[x(5x - 4) + 3] - 2\} + 1$

original expression:	$5x^4$	4 multiplications x 10 =	40 time units
	$4x^3$	3 multiplications x 10 =	30 time units
	$3x^2$	2 multiplication x 10 =	20 time units
	$2x$	1 multiplication x 10 =	10 time units
		4 additions and subtractions =	4 time units
	Total		**104 time units**

factored expression	$x\{x[x(5x - 4) + 3] – 2\} + 1$		
		4 multiplications x 10 =	40 time units
		4 additions =	4 time units
	Total		**44 time units**

The factored expression takes less than half the time to evaluate.

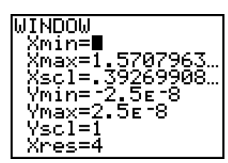
2. Using a graphics calculator, set

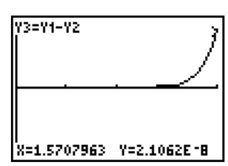
$$Y1 = 1 - 0.499999996x^2 + 0.041666641x^4 - 0.001388838x^6 + 0.000024760x^8 - 0.000000260x^{10}$$

$Y2 = \cos x$

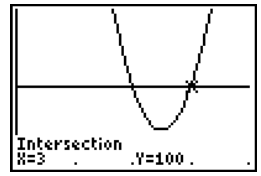
$Y3 = Y1 - Y2$

The graph of Y3 (shown alongside) shows that the maximum error of 2.1062×10^{-8} occurs at $x = \frac{\pi}{2}$.





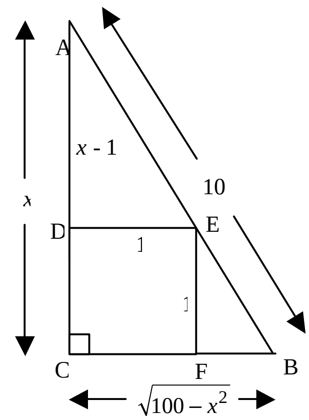
3. A sketch of the two functions shows that the volume of water is less or equal to 100 ML for x in [2,3]. Taking January 1 as $t = 0$, then $x = 2$ represents March 1 and $x = 3$ represents April 1.



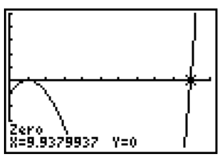
Hence irrigation was not permitted during the month of March.

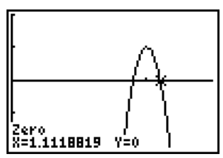
4. Since ΔABC is a right-angled triangle, by Pythagoras' Theorem, BC has length $\sqrt{100-x^2}$.
Since ΔABC is similar to ΔAED,

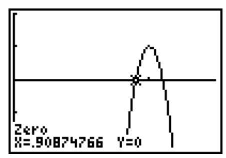
$$\frac{\sqrt{100-x^2}}{x}=\frac{1}{x-1} \qquad \text{\{property of similar }\Delta\text{s\}}$$
$$(x-1)\sqrt{100-x^2}=x \qquad \text{\{multiply both sides by } x(x-1)\text{ \}}$$
$$(x-1)^2(100-x^2)=x^2 \qquad \text{\{square both sides\}}$$
$$(x^2-2x+1)(100-x^2)=x^2 \qquad \text{\{expand and simplify\}}$$
$$100x^2-x^4-200x+2x^3+100-x^2=x^2$$
$$x^4-2x^3-98x^2+200x-100=0$$



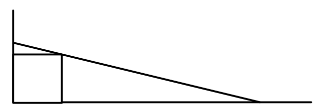
We find the roots using a graphics calculator. The first screen shot below shows that one root is 9.938 metres, and from the diagram, this is the one we are seeking. But there are two other positive roots:
$x = 1.112$ and $x = 0.9087$.







Ah, yes! The first of these is another valid solution, as the diagram alongside shows. There are two ways to rest the ladder against the cube.



The other root, while it is a root to the 4th degree polynomial, is not a solution to our problem, as the ladder cannot touch the wall at a point that is less than 1 metre above the ground.

5. a. At the end of the second year, the amount in the bank is the amount at the start of the year multiplied by $1 + i$. Algebraically it is:
$$(3000+7000x)x=3000x+7000x^2$$

b. Continuing the pattern:

At the start of the third year:	$4500+3000x+7000x^2$
At the end of the third year:	$(4500+3000x+7000x^2)x=4500x+3000x^2+7000x^3$
At the start of the fourth year:	$4000+4500x+3000x^2+7000x^3$
At the end of the fourth year:	$4000x+4500x^2+3000x^3+7000x^4$
At the start of the fifth year:	$9000+4000x+4500x^2+3000x^3+7000x^4$
At the end of the fifth year:	$9000x+4000x^2+4500x^3+3000x^4+7000x^5$

We want this investment to total \$35 000, so we need to solve:
$9x+4x^2+4.5x^3+3x^4+7x^5=35$ {dividing both sides by 1000}
The graphics calculator solution is shown alongside.
Since $1 + i \cong 1.086$, you require a return of about 8.6% per annum.