The Monty Hall Problem

One of the most intriguing problems in introductory probability is the Monty Hall problem. It goes like this:

> *Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the other doors, opens another door, say No. 3, which has a goat. He then says to you, 'Do you want to switch to Door No. 2?'*
>
> *Is it to your advantage to take the switch?*

Many Sunday newspapers in the USA include a full colour supplement called Parade magazine. A regular feature of Parade is a column by Marilyn Von Savant, who is (or was) listed in the Guinness Book of World Records Hall of Fame for "Highest IQ". The question above was sent to Von Savant, who in her column gave the answer "You should switch."

Her answer prompted debate in the hallowed halls of universities across America, at the Central Intelligence Agency and at the Los Alamos National Laboratory in New Mexico. Von Savant claims to have received over 10 000 letters, most disagreeing with her answer. Over 1 000 of these letters came from people with PhDs, and many of those were professional mathematicians and scientists.

One such letter read "As a professional mathematician, I'm very concerned with the general public's lack of mathematical skills. Please help by confessing your error and, in the future, being more careful." Another read "You are utterly incorrect. How many irate mathematicians are needed to get you to change your mind?". Paul Erdos, a highly respected and prolific modern-day mathematician who has authored or co-authored over 200 papers on mathematics, disagreed with Von Savant's answer when first presented with the problem.

Most of those that disagreed believed that once a door has been opened, and shown to house a goat, then the remaining doors each have a 50:50 chance of hiding the prize. But to the astonishment, and the embarrassment, of these mathematicians, Von Savant was right – your chance of winning the prize is better if you switch.

Over the years there have been many attempts to explain why to a sceptical audience. Here is our preferred explanation. There are 3 possible outcomes – My door (Door 1, say) hides the prize, the door the host opens (Door 3, say) hides the prize and the remaining door (Door 2, say) hides the prize.

Since these are mutually exclusive we can write:

P(Door 1 hides the prize) + P(Door 3 hides the prize) + P(Door 2 hides the prize) = 1

Now

P(Door 1 hides the prize) = $\frac{1}{3}$, since I chose the door at random from 3 doors.

P(Door 3 hides the prize) = 0, since the host never opens a door that hides the prize.

Substituting

P(Door 1 hides the prize) + P(Door 3 hides the prize) + P(Door 2 hides the prize) = 1

$\therefore$ $\frac{1}{3}$ + 0 + P(Door 2 hides the prize) = 1

Hence

P(Door 2 hides the prize) = $\frac{2}{3}$

Therefore, you should switch to Door 2. You will win the prize $\frac{2}{3}$ of the time!

Here is a shorter way of saying the same thing: The chance I selected the correct door is $\frac{1}{3}$. Opening a door and revealing a goat doesn't change that. Hence the chance that the prize is behind the other door must be $\frac{2}{3}$.

But even with such crystal-clear logic, I guarantee that not everyone will be convinced! Are you?