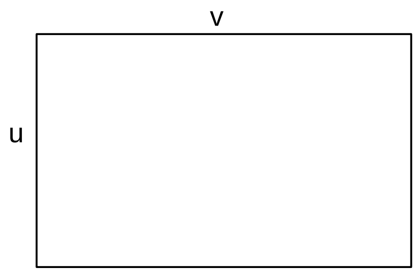
©2001. Kiddy Bolger, Rex Boggs, Rhonda Faragher, John Belward

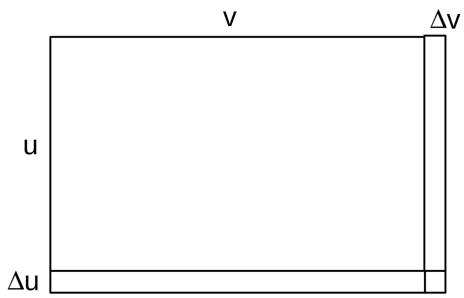
The Product Rule

The product rule is suggested by the following.

Consider a rectangle with sides of lengths *u* and *v*.



We can make a small change to the area of this rectangle by increasing its length by Δv and its width by Δu.



The small change in the area is given by

$$\Delta(uv) = v\Delta u + u\Delta v + \Delta u\Delta v$$

{since $A = uv$, a small change in area is given by $\Delta(uv)$}

If we imagine the area changing over time t, then we have

$$\frac{\Delta(uv)}{\Delta t} = \frac{v\Delta u}{\Delta t} + \frac{u\Delta v}{\Delta t} + \frac{\Delta u\Delta v}{\Delta t}$$

Now letting Δt approach 0, we have

$$\lim_{\Delta t\to 0}\frac{\Delta(uv)}{\Delta t} = \lim_{\Delta t\to 0}\frac{v\Delta u}{\Delta t} + \lim_{\Delta t\to 0}\frac{u\Delta v}{\Delta t} + \lim_{\Delta t\to 0}\frac{\Delta u\Delta v}{\Delta t}$$

And taking the limit gives

$$\frac{d(uv)}{dt} = v\frac{du}{dt} + u\frac{dv}{dt} + \frac{du}{dt}0$$

{If $\Delta t \to 0$ then $\Delta v \to 0$, hence the last term $\to 0$}

$$= v\frac{du}{dt} + u\frac{du}{dt}$$

This suggests that the rule for differentiating a product is given by:

$$\frac{d(uv)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$