Mathematics for Queensland, Year 12
Worked Solutions to Exercise 1H

Hints

1. For an even function, $f(-x) = f(x)$. Algebraically this means that the value of the function is the same for any value *x* and its negative. Graphically this means that the function is symmetric about the *x*-axis. For an odd function, $f(-x) = -f(x)$. Graphically this means that the function has rotational symmetry about the origin through 180^0.

   Also, consider the values of sin, cos and tan in the unit circle.

2. This can be done most easily using a graphics calculator. The absolute value function on a TI-82 or TI-83 calculator is found under [Math] [Num] [Abs].

3. The scale can be determined from the diagram. The answer depends upon where we place the origin. You get to choose, so choose a nice spot. Find the period and amplitude, and you are well underway!

4. The function is of the form $y = A\cos[B(x - C)] + D$. We need to determine the value of each of these parameters from the information given. Since C depends upon the date, and the starting date doesn't matter, you can choose C to be any value you wish. Which value is easiest?

5. This is best solved using a graphics calculator.

6. a. What is the maximum value of sin *nt*, for any value of *n*? Use this information to solve for *a* when $p = 180$.

   b. This is very difficult to solve algebraically. Try guess-check-refine.

7. This is best solved using a graphics calculator. How many minutes after midnight is 6:15 a.m.?

Mathematics for Queensland, Year 12
Worked Solutions to Exercise 2J

1. For an even function, $f(-x) = f(x)$. Algebraically this means that the value of the function is the same for any value *x* and its negative. Graphically this means that the function is symmetric about the *x*-axis.

   Since the graph of the cosine function is symmetric about the *x*-axis, it is an even function.

   Also, from the unit circle, it should be clear that cos(*x*) = cos(-*x*).

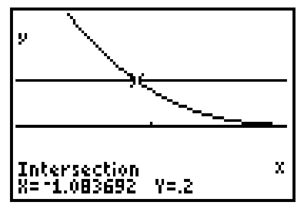
   For an odd function, $f(-x) = -f(x)$. Graphically this means that the function has rotational symmetry about the origin through 180^0. The graph of both the sine function and the tangent function have this symmetry and hence are odd functions.

   Also, from the unit circle, it should be clear that sin(-*x*) = -sin(*x*) and tan(-*x*) = -tan(*x*).

2. This can be done most easily using a graphics calculator. The absolute value function on a TI-82 or TI-83 calculator is found under [Math] [Num] [Abs]. We will work in radians.

   $Y1 = |\sin x - x|$

   $Y2 = 0.2$

   From the graph, one solution is:

   $x = -1.08$

   The absolute value makes the graph is symmetric about the x-axis, so the second solution is:

   $x = 1.08$

3. The answer depends upon where we place the origin. The scale can be determined to be 1:10 from the diagram. From the scale given, the period is $\frac{2}{3} \times 460 \approx 307$. By measuring and applying the scale, we find the amplitude is approximately 50 cm. If we place our origin at the centre of the right side, then we have a cosine function of the form:

   $y = A\cos(Bx)$

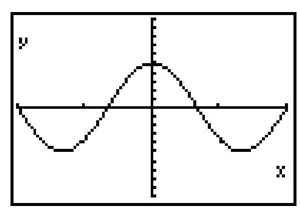
   where

   $A = 50$

   and $B = \frac{2p}{\text{period}} = \frac{2p}{307} \approx 0.0205$

   Hence our function is given by

   $y = 50\cos(0.0205x)$

   As a check, we will sketch this function in the window (-230, 230, 115; -100, 100, 10). The graph is alongside. Looks pretty good!

4. The function is of the form $y = A\cos[B(x - C)] + D$. We need to determine the value of each of these parameters.

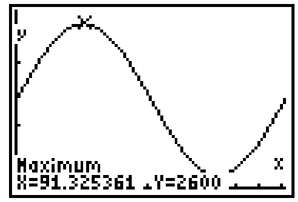
   The amplitude $A = ½(Y_{max} - Y_{min}) = 0.5(2600-200) = 1200$.

   Since the average flow = 200 + 1200 = 1400, the vertical shift *D* = 1400 units.

   C depends on the date from which we first calculate flow. Since the date isn't specified, we can set *C* = 0.

The period is one year or 365 days, so

$$B = \frac{2p}{\text{period}} = \frac{2p}{365} \approx 0.0172$$



Hence our function is given by:

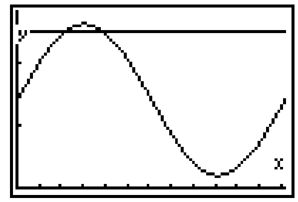
$$= 1200\sin(0.0172\ \ ) + 1400$$

The graph is shown alongside.

We now add the function

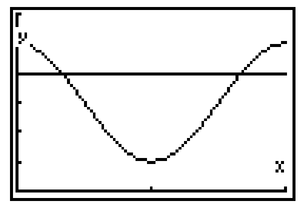
Y2 = 2500

and use our calculator to find the values of for which the function is greater than 2500.



The graphs intersect at x = 67 and
to our model, the flow is greater than 2500 units for about 115-

5. s shown alongside using the window (0,12,1; 0,6,1).



From the graphics calculator, the -coordinates of the intersection are:

$x = 2$

and = 10.

Therefore we expect the water to be at the 4 metre mark at 2 a.m. and 10 a.m.

6. Given $p =\ \ + a\ \ t$. If p = 180 then

$$180 = 125 + a\sin 10t$$

$$55 = a\sin 10t$$

Now sin 10t has a maximum value of 1, therefore the artificial artery will rupture is the value of a is greater than 55.

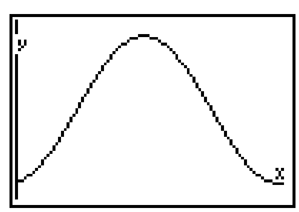
b. Given is $p = 125 + a\sin 10t$. We need to find the value of a such that the value of the function exceeds 150 for more than 0.25 seconds. This is difficult to solve algebraically, so we will solve this problem using guess-check-refine.

We will make a table to keep track of our results.

a	time > 150
55	0.2670 - 0.0471 = 0.2199
60	0.2712 - 0.0430 = 0.2282
80	0.2824 - 0.0318 = 0.2506

Using this criterion, the blood vessel will rupture if the exertion level exceeds 80.

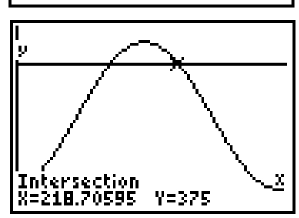
7. The graph of $m = 340 - 50\cos[(n+10)\times \frac{2p}{365}]$ in the window (0,365,0; 280,400,0) is shown alongside.



Now 6:15 a.m. is 6 x 60 + 15 = 375 minutes after midnight, so we need to add Y2 = 375 to the graph. The points of intersection occur at

x = 126

and x = 218



Hence, you leave for work *before* sunrise from day 127 to day 217, for a total of 217 – 127 + 1 = 91 days.