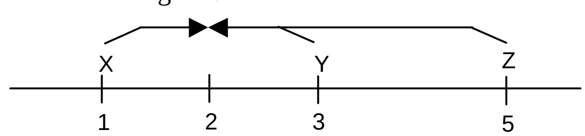
Mathematics for Queensland, Year 12
Worked Solutions to Exercise 7M

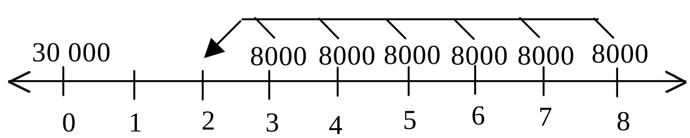
Hints

1. Draw a time value diagram.



2. You can think of this as a single payment of $50 000 now, plus the present value of an ordinary annuity with 19 payments of $50 000 at the end of each time period.

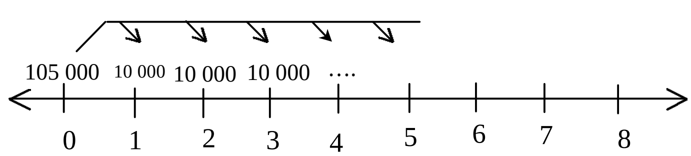
3. First find the time value of the six semi-annual payments on 1 January 2004 (time 2 on the diagram). Then find the time value of this present value on 1 January 2003.



4. By her 30th birthday, Danielle had made 120 payments of $200. Find the future value of this annuity on her 30th birthday. What does this then grow to by age 55?

   What monthly payment must Nathan make so his annuity grows to the same amount?

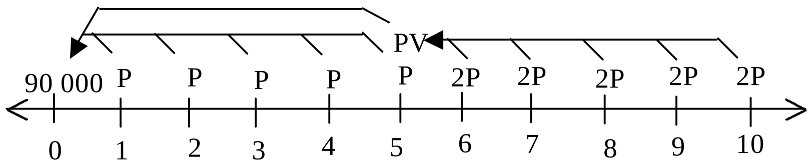
5. Use 1 year as our calculation period. Find the future value of 12 monthly deposits. Then find the weekly deposit that results in the same amount at the end of the year.

6. 

   After the $15 000 lump sum is paid, the remaining $105 000 is the present value of an ordinary annuity. Find the number of payments.

   How much of the original $105 000 is needed to pay for these payments? How much of the $105 000 is left over? What is its value when the last payment is to be made?

7.



Step 1: Find the present value of the final 5 payments of $2P each at time *t* = 5, in terms of P.

Step 2: Find the time value of this single amount at time t=0.

Step 3: The present value of the first 5 payments of $P each is just half of that found in Step 1.

Step 4: Add these amounts, set the sum equal to $90 000 and solve for P.

8. a. Find the monthly repayments on a 30 year loan at 8% p.a. compounded monthly. Now find the interest using the simple interest formula.

b. After 5 years, how many years left to pay? How much is still owed?

9. (Optional topic) Track the loan for the first few months, to see the pattern.

The Interest is the simple interest on the Balance for 1 month at 8% p.a.

The New Balance = Balance – Repayment + Interest.

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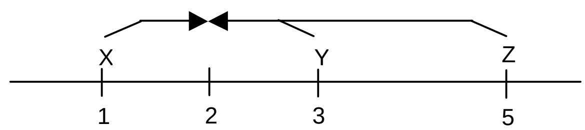
1. Draw a time value diagram.



We need to find the time value (TV) of each of these payments at two years.

X: $TV_X = 6300(1+0.115)^1 = 7024.50$

Y: $TV_Y = 3200(1+0.115)^{-1} = 2869.96$

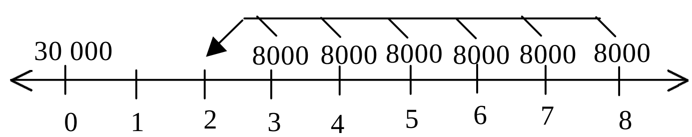
Z: $TV_Z = 5750(1+0.115)^{-3} = 4148.04$

The total owing in two years time is \$7024.50 + \$2869.96 + \$4148.04 = \$14 042. 50.

2. We will view this as a single payment of \$50 000 now, plus the present value of an ordinary annuity with 19 payments of \$50 000 at the end of each time period. It could also be viewed as the present value of an annuity due, with 20 payments.

Using the TVM Solver of a TI-83,
the present value of the ordinary annuity = \$656 696.97.
Hence the actual value as a lump sum today =
\$656 696.97 + \$50 000 = \$706 696.97

3. First find the time value of the six semi-annual payments on 1 January 2004 (time 2 on the diagram). Then find the time value of this present value on 1 January 2003.



Using the TVM Solver of a TI-83,
the present value of the 6 payments at time t=2 is \$39 964.24.

Using the compound interest formula, find the value of \$39 964.24 at time t = 0:

$$A = P(1+i)^n$$

$$P = \frac{A}{(1+i)^n}$$

$$= \frac{39\,964.24}{1.055^2}$$

$$= 35\,905.97$$

Add in the initial deposit
PV = 35 905.97 + 30 000 = 65 905.97.

The cash price of the business on 1 January 2003 is \$65 905.97.

4. The question is ambiguous, so you will need to state your assumptions.

The answers below assume that Danielle makes 120 deposits starting at the end of her birth month and finishing at the end of the month prior to her 30th birthday (i.e. an ordinary annuity).

The answers in brackets assume that she makes 120 deposits, starting at the beginning of her birth month with the last payment at the beginning of the month prior to her 30th birthday. She then leaves the money in the account until the start of her birth month (i.e. an annuity due).

Surprisingly, the calculation of Nathan's monthly savings is the same either way!

N.B. Other interpretations of the question are possible and most likely are correct if you STATE YOUR ASSUMPTIONS.

By her 30th birthday, Danielle had made 120 payments of \$200. Using the TVM solver, the future value of this annuity on her 30th birthday is

FV = \$36 589.21 (\$36 833.14)

Using the compound interest formula, this grows to

A = 268 571.24 (\$270 361.73)

at age 55.

Nathan made 300 payments of \$P. For this to grow to \$268 571.24 (\$270 361.24), using the TVM solver, each payment must be

P = \$282.40

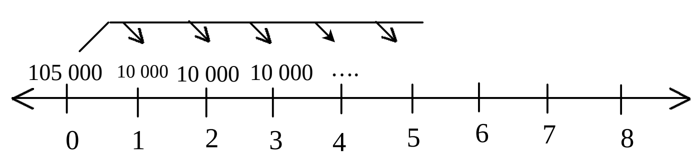
Nathan must make 300 payments of \$282.40 to match Danielle's 120 payments of \$200. It pays to start saving early!

5. We will use 1 year as our calculation period.

Using the TVM calculator, the 12 monthly deposits of \$1000 have a future value after 1 year of \$12 594.68.

Using the TVM calculator, for 52 weekly deposits of \$P to have a future value after 1 year of \$12 594.68, each payment must be \$231.10.

6.



After the \$15 000 lump sum is paid, the remaining \$105 000 is the present value of an ordinary annuity. Using the TVM solver, the number of payments is 17.

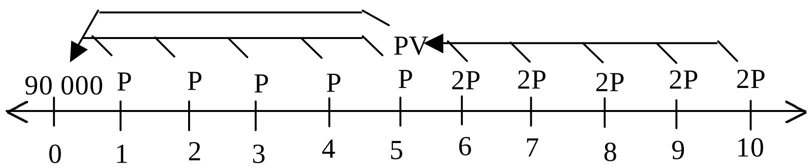
Now the PV of these 17 payments is \$104 772.60. There is still \$105 000 - \$104 772.60 = \$227.40 of the pension at time 0 that is not used for the payments.

To find the value of the final payment, we use the compound interest formula to find the value of \$227.40 after 18 years:

$$A = 227.40(1.06)^{18} = 649.08$$

Mrs Dickenson's final payment in the 18th year will be \$649.08.

7.



Step 1: Find the present value of the final 5 payments of $2P each at time t = 5, in terms of P:

$$A = \frac{R[1-(1+i)^{-n}]}{i}$$
$$= \frac{2P(1-1.09^{-5})}{0.09}$$
$$= 2P \times 3.890$$
$$= 7.780P$$

Step 2: Find the time value of this amount at time t=0, using the compound interest formula.

$$PV = FV(1+i)^{-n}$$
$$= 7.78P(1.09)^{-5}$$
$$= 5.056P$$

Step 3: The present value of the first 5 payments of $P each is just half of that found in Step 1, or $A = 3.890P$

Step 4: Add these amounts, set the sum equal to $90 000 and solve for P.

$5.056P + 3.890P = 90\ 000$
$8.946P = 90\ 000$
$P = 10\ 060.36$

She must make 5 payments of $10 060.36, followed by 5 payments of $20 120.72.

8. a. Use the TVM Solver to find the monthly repayments on a 30 year loan at 8% p.a. compounded monthly:

R = $880.52

Use the simple interest formula to find the interest owed for one month on a loan of $120 000 at 8% p.a.:

I = Prt = 120 000 $\times$ 0.08 $\times$ $^{1}/_{12}$ = $800

The interest only loan is $880.52 - $800 = $80.52 less per month.

b. After 5 years on an interest only loan, the couple still owe $120 000, but now have only 25 years to repay the loan. Using the TVM calculator, the repayment is:

R = 926.18

The repayments will increase by $926.18 – $800 = $126.18 per month, assuming that the interest rate remains at 8%.

9. (Optional topic) Lets track the loan for the first few months, to see the pattern.

The Interest is the simple interest on the Balance for 1 month at 8% p.a.

The New Balance = Balance – Repayment + Interest.

Time Period	**Balance**	**Repayment**	**Interest**	**New Balance**
1	$120 000.00	$926.00	$800.00	$119 874.00
2	$119 874.00	$926.00	$799.16	$119 747.16
3	119 747.16	$926.00	$798.31	$119 619.47
…				

The recursive function for the Balance $B(n)$ is given as follows:
$B(n)$ = (Old Balance) – Repayment + Interest

Algebraically,

$$B(n) = B(n-1) - 926 + \frac{0.08 \times B(n-1)}{12}$$